



Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the big- ger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17 10 11 12 13 14 15 16 17 18 19 20 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. This is an essential skill for column addition later.	Start with the bigger number and use the smaller number to make 10. Use ten frames.	3 + 9 = Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 9 + 5 = 14	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	2 more than 5.	Cross 2 mare hats 5 + 2 =	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

ADDITION 4



	Pictorial	Abstract
50= 30 = 20		20 + 30 = 50
11111		70 = 50 + 20
	3 tens + 5 tens = tens 30 + 50 =	40 + □ = 60
Model using dienes and bead strings	Use representations for base ten.	
Children explore ways of making numbers within 20	20	1 + 1 = 16
	+ = 20 20 - =	
	$\begin{array}{cccc} \begin{array}{cccc} & & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{array} & \begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & \\ & & \\ $	3 + 4 = 7
חחח חחח חחחחחח	+ =	leads to
+ =		30 + 40 = 70
		leads to
	Children draw representations of H,T and O	300 + 400 = 700
	英英英英英英英 著 著 著	23 25
2.4.7		?
3 + 4 = 7	7 + 3 = 10	23 + 25 = 48
	Model using dienes and bead strings Children explore ways of making numbers within 20 ———————————————————————————————————	Model using dienes and bead strings Children explore ways of making numbers within 20 The product of the prod

ADDITION 4



Objective &	Concrete	Pictorial	Abstract
Strategy			
Add a two digit number and ones	17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 17 + 5 = 22 3 2 16 + 7 16 + 7 16 20 23	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17
Add a 2 digit num- ber and tens	25 + 10 = 35 Explore that the ones digit does not change	27 + 30 +10 +10 +10 	27 + 10 = 37 27 + 20 = 47 27 + = 57
Add two 2-digit numbers	Model using dienes , place value counters and numicon	47 67 72 47 67 70 72 Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 = 12 60 + 12 = 72
Add three 1-digit numbers	Combine to make 10 first if possible, or bridge 10 then add third digit	Regroup and draw representation.	4+7+6 = 10+7 = 17 Combine the two numbers that make/ bridge ten then add on the third.



Objective &	Concrete	Pictorial	Abstract
Strategy Column Addition—no regrouping (friendly numbers)	T O Model using Dienes or numicon	Children move to drawing the counters using a tens and one frame.	2 2 3
Add two or three 2 or 3-digit numbers.	Add together the ones first, then the tens. Tens Units 45 34 7 9 Calculations 21+42 = 142 Move to using place value counters	tens ones	+ 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	Exchange ten ones for a ten. Model using numicon and pv counters. 146 +527	Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line	$\begin{array}{cccccccccccccccccccccccccccccccccccc$



Objective & Strategy	Concrete					Pict	orial		Abstract
Y4—add numbers with up to 4 digits	counters to a a ten and ter	etinue to use diene add, exchanging to n tens for a hundre r a thousand.	en ones for	Draw	7	1	5	1	3517 + 396 3913 Continue from previous work to carry hundreds as well as tens.
Y5—add numbers with more than 4 digits. Add decimals with 2 dec- imal places, including money.		tenths tenths cimal place value of		2 tens	00	1.79	+ents	hundred the	Relate to money and measures. 72.8 +54.6 127.4 1 1
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5			As Y5					8 1, 0 5 9 3, 6 6 8 15, 3 0 1 + 20,5 5 1 1 2 0,5 7 9 1 1 1 1 2 3 · 3 6 1 9 · 0 8 0 5 9 · 7 7 0 + 1 · 3 0 0 9 3 · 5 1 1



Taking away ones. Use physical objects, counters, cubes etc to show how objects can be taken away. 6-4 = 2 15-3 = 12 Cross out drawn objects to show what has been taken away. Counting back Counting back Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards. Count back in ones using a number line. Compare objects and amounts T am 2 years older than my sister. Count on using a number line to find the difference. Count on using a number line to find the difference.	Objective & Strategy	Concrete	Pictorial	Abstract
Counting back Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards. Count back in ones using a number line. Compare objects and amounts Count on using a number line to find the difference. Count on using a number line to find the difference. Count on using a number line to find the difference.		to show how objects can be taken away. $6-4=2$		
Move the beads along the bead string as you count backwards. Count back in ones using a number line. Find the Difference T am 2 years older than my Count back in ones using a number line. Count back in ones using a number line. Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?	Counting back	Move objects away from the group,		
Count on using a number line to find the difference. Count on using a number line to find the difference. Tam 2 years older than my		Move the beads along the bead string as you count	Count back in ones using a number line.	
		7 'Seven is 3 more than four' 4 'I am 2 years older than my		

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	Use pictorial representations to show the part.	Move to using numbers within the part whole model. 5
Make 10	Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	5-2 = 3	《	8 2 10 = 8 + 2 10 = 2 + 8 10-2 = 8 10-8 = 2



Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	90000 90000 20 - 4 =	20—4 = 16
Partitioning to sub- tract without re- grouping. 'Friendly numbers'	Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off. 43—21 = 22	43—21 = 22
Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.	34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	76 80 90 93 'counting on' to find 'difference' Use a number line to count on to next ten and then the rest.	93—76 = 17

Y2

SUBTRACTION



Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	Use base 10 or Numicon to model	Darw representations to support understanding	$47 - 24 = 23$ $-\frac{40 + 7}{20 + 3}$ Intermediate step may be needed to lead to clear subtraction understanding.
Column subtraction with regrouping	Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	Tens Ones 29 10 10 + 6 = 10 Children may draw base ten or PV counters and cross off.	836-254*582 Begin by partitioning into pv columns 728-582=146 728-582=146 728-582=146 74 12 8 75 8 2 14 6



Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. Introduce decimal subtraction through context of money	234 - 179 O O O O O O O O O O O O O O O O O O O	Children to draw pv counters and show their exchange—see Y3	2 X 5 4 - 1 5 6 2 1 1 9 2 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 dig- its, including money and measures. Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal	As Year 4	Children to draw pv counters and show their exchange—see Y3	3
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			**************************************





Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manip- ultives including cubes and Numicon to demonstrate doubling	Draw pictures to show how to double numbers	Partition a number and then double each part before recombining it back together.
	double 4 is 8 4 × 2 = 8	Double 4 is 8	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
Counting in multi- ples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	x = 8 Use manipulatives to create equal groups.	Draw to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Objective &	Concrete	Pictorial	Abstract
Strategy Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 = 15	Write addition sentences to describe objects and pictures. $2+2+2+2=10$
Understanding ar- rays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show under- standing	3 x 2 = 6 2 x 5 = 10

Objective &	Concrete	Pictorial	Abstract
Strategy			
Doubling	Model doubling using dienes and PV counters. 40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 16 10 10 1 12 20 12 20 12 21 22
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 5+5+5+5+5+5+5+5=40	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 3 3 3 3 3	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30

Objective &	Concrete	Pictorial	Abstract
Strategy			
Doubling	Model doubling using dienes and PV counters. 40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 16 10 6 1x2 1 x2 20 + 12 = 32
Counting in multi-	Count the groups as children are skip	Number lines, counting sticks and bar	Count in multiples of a number aloud.
ples of 2, 3, 4, 5, 10 from 0 (repeated addition)	counting, children may use their fingers as they are skip counting. Use bar models. 5+5+5+5+5+5+5+5+5=40	models should be used to show representation of counting in multiples. 3 3 3 3 3	Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 4 × 3 =

3

Objective &

Strategy

Counting in multi-

ples of 2, 3, 4, 5, 10

(repeated addition)

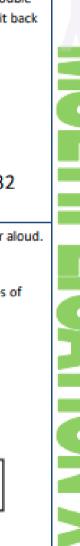
from 0

Doubling

Westcliff P	rimary Academy Calculations Polic		
Concrete	Pictorial	Abstract	
Model doubling using dienes and PV counters. 40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 16 10 6 1x2 1 x2 20 + 12 = 32	
Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30	

3

3



Y2



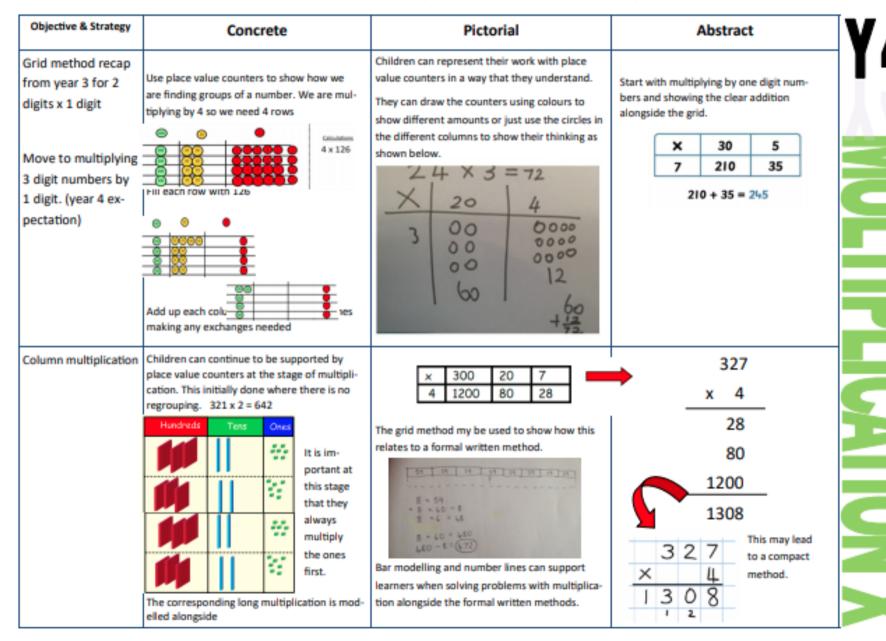
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
ing the Inverse is should be ught alongside vision, so pupils arn how they ark alongside ch other.		X	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.



Objective & Abstract Concrete Pictorial Strategy Children can represent their work with place Grid method Show the links with arrays to first intro-Start with multiplying by one digit numvalue counters in a way that they understand. duce the grid method bers and showing the clear addition alongside the grid. 4 rows They can draw the counters using colours to of 10 show different amounts or just use the circles in 4 rows 30 5 the different columns to show their thinking as 35 210 shown below. Move onto base ten to move towards a X 3 = 72 210 + 35 = 245more compact method. 4 rows of 13 20 00 Moving forward, multiply by a 2 digit number 0000 0000 showing the different rows within the grid 00 Move on to place value counters to show 0000 method. 00 how we are finding groups of a number. We are multiplying by 4 so we need 4 rows 10 8 Calculations 4 x 126 100 80 10 Bar model are used to explore missing numbers 30 24 Fill each row with 126 Calculations 4×126 = 20 Add up each column, starting with the ones making any exchanges needed Then you have your answer.

Y3





Objective &	Concrete	Pictorial	Abstract	V 6
Strategy Column Multiplication for 3 and 4 digits x 1 digit.	Hundreds Tens Ones It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	× 300 20 7 4 1200 80 28	327 x 4 28 80 1200 1308 3 2 7 X 4 1308 This will lead to a compact method.	
Column multiplication	Manipulatives may still be used with the cor- responding long multiplication modelled alongside.	Continue to use bar modelling to support problem solving	1 8 18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3) 2 3 4 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first 1 2 3 4 0 (1234 x 6) 1 9 7 4 4	



Objective &	Concrete	Pictorial	Abstract
Strategy			
Multiplying decimals			Remind children that the single digit belongs
up to 2 decimal plac-			in the units column. Line up the decimal
es by a single digit.			points in the question and the answer.
			3 · 1 9
			× 8
			25.52
			7

Objective &	Concrete	Pictorial	Abstract
Strategy			
Division as sharing		Children use pictures or shapes to share quanti- ties.	12 shared between 3 is
Use Gordon ITPs for	00	\$ \$	4
modelling	SCHOOL SERVICE STATE OF THE PARTY OF THE PAR	\$ \$	
		8 Shareu between z is 4	
		Sharing:	
		12 shared between 3 is 4	
	10		
	have 10 cubes, can you share them equally in		
	groups?		

Objective & Strategy			Abstract
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 + 2 = 4 Children use bar modelling to show and support understanding.	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Think of the part as a whole spin to the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

20 ÷ 5 = ? 5 x ? = 20

Objective &	Concrete	Pictorial	Abstract
Strategy Division as grouping	division problems. 24?		How many groups of 6 in
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4



Objective &	Concrete	Pictorial	Abstract
Strategy			
Division with remainders.	Divide objects between groups and see how much is left over Example without 40 + 5 Ask "How many Example with re 38 + 6 For larger numbe jumps can be rec	5s in 40? 5+5+5+5+5+5+5 = 8 f 0 5 10 15 20 25 30 35 40 mainder.	a remainder of 2

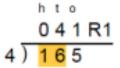
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	2	1	8			VA.R
4	8	7	2			
love onto						
Г	8	3	r	2		
5	4 3	2				
inally mov		cimal	places t	to div	ide the	
	5 5	1	4		6	
3 5	5 5	1	1		0	

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	3 2 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 8 7 2 Move onto divisions with a remainder. 8 6 r 2 5 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 6 16 21 3 5 5 1 1 . 0

Long Division

Step 1—a remainder in the ones



- 4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).
- 4 goes into 16 four times.
- 4 goes into 5 once, leaving a remainder of 1.

- 8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).
- 8 goes into 32 four times (3,200 + 8 = 400)
- 8 goes into 0 zero times (tens).
- 8 goes into 7 zero times, and leaves a remainder of 7.

Y6

Long Division

Step 1 continued...

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

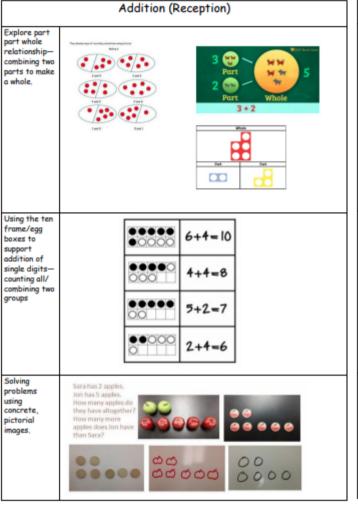
Long Division

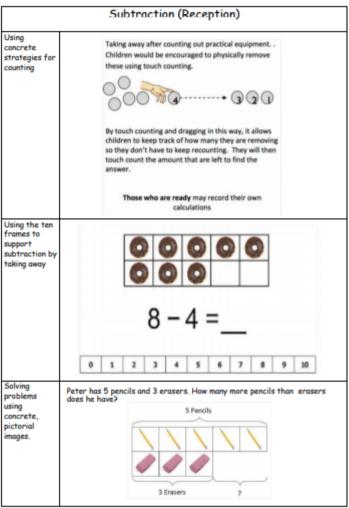
Step 2-a remainder in any of the place values

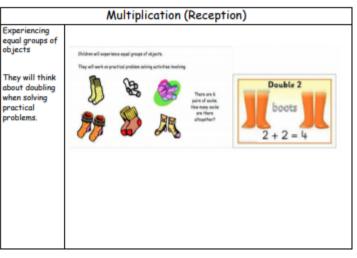
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.	
1 2)278	2) 2 7 8 -2 0	1 8 2) 2 7 8 -2 0 7	
Two goes into 2 one time, or 2 hundreds + 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.	
Divide.	Multiply & subtract.	Drop down the next digit.	
2) 2 7 8 -2 0 7	13 2)278 -2 07 -6 1	2) 2 7 8 -2 0 7 -6 1 8	
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.	
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.	
139 2)278 -2 07 -6	139 2)278 -2 07 -6 18 -18	2)278 -207 -207 -6 18 -18	
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 x 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.	











Sharing

practical

objects.

Hearing and being exposed

to language to

describe half and seeing visual

representations

